Journal of the Korean Data & Information Science Society
2019, **30**(3), 703–711

http://dx.doi.org/10.7465/jkdi.2019.30.3.703
한국데이터정보과학회지

Approximate maximum product spacing estimation of half logistic distribution under progressive type II censored samples[†]

Kyeongjun Lee[1]

[1]Dvision of Mathematics and Big Data Science, Daegu University

Received 6 April 2019, revised 24 April 2019, accepted 24 April 2019

Abstract

In most of the life testing and reliability experiments, the experimenter is often, unable to observe life time of all items put on test and the data avilable to the experimenter is censored data. Under classical estimation set up, the maximum product spacings method is quite effective and several authors advocated the use of this method as an alternative to MLE, and found that this estimation method provides better estimates than MLE in various situations. In this paper, we derive the MPSE for the parameter and reliability function of half-logistic (HL) distribution. And we derive the approximate MPSE for the parameter and reliability function of HL distribution using Talyor series expansion. We also compare the proposed estimators in the sense of the root mean squared error (RMSE) and bias for various progressive type II censored samples. In addition, real data example based on progressive type II censoring scheme have been also analysed for illustrative purposes.

Keywords: Approximate maximum product spacings estimation, half-logistic distribution, maximum product spacings estimation, progressive type II censoring.

1. Introduction

In most of the life testing and reliability experiments, the experimenter is often, unable to observe life time of all items put on test and the data available to the experimenter is censored data. In fact, in life testing and reliability experiments, censoring occurs in a natural way. Various type of censoring schemes along with heir advantages and disadvantages are available in the literature. Among these censoring schemes, now a days, progressive type-II censoring scheme is gaining rapid popularity. This censoring scheme was introduce by Cohen (1963) and has been widely discussed in literature by several authors. AL-Hussaini *et al.* (2015) derive the one-sample Bayes prediction interval based on progressive type II censored samples from the half-logistic (HL) distribution. Lee and Cho (2017) derive

[†] This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2017R1C1B5018126).
[1] Associate professor, Division of Mathematics and Big Data Science, Daegu University, Gyeongsan 38453, Korea. E-mail: indra_74@naver.com

the exact inference for competing risks model with generalized progressive hybrid censored sample from an exponential distribution. Yun and Lee (2018) proposed the goodness of fit tests for progressive type II censored data from a Gumbel distribution.

Under classical estimation set up, the maximum product spacings method is quite effective and several authors advocated the use of this method as an alternative to MLE, and found that this estimation method provides better estimates than MLE in various situations. The maximum product spacings method was introduced by Cheng and Amin (1983). Maximum product spacings method is most suitable method, especially to those cases where one of the parameter have an unknown shifted origin. It is also observed that maximum product spacings estimators (MPSE) possesses almost all properties being possessed by MLE. Rahman *et al*. (2007) consider the method of product of spacings in the two-parameter gamma distribution. Singh *et al*. (2014) consider the traditional estimation method and maximum product spacing method in generalised inverted exponential distribution. Singh *et al*. (2016) consider the maximum product spacings method for the estimation of parameters of generalized inverted exponential distribution under progressive type II censoring.

The HL distribution is obtained by folding the logistic distribution. Balakrishnan (1985) proposed the order statistic from the HL distribution as lifetime model. The random variable X has a HL distribution if it has a probability density function (pdf) and cumulative distribution function (cdf) of the form:

$$
\begin{aligned}
g(x;\sigma) &= \frac{2\exp\left(-\frac{x}{\sigma}\right)}{\sigma\left[1+\exp\left(-\frac{x}{\sigma}\right)\right]^2}, \\
G(x;\sigma) &= \frac{1-\exp\left(-\frac{x}{\sigma}\right)}{1+\exp\left(-\frac{x}{\sigma}\right)} \quad x>0,\ \sigma>0
\end{aligned}
$$

and reliability function of the form

$$
\Re(t) = \frac{2\exp(-t/\sigma)}{1+\exp(-t/\sigma)},
$$

where σ is the scale parameter. Let the random variable $Z = X/\sigma$, the pdf and cdf of the random variable Z with the standard HL distribution are given by

$$
f(z;\sigma) = \frac{2\exp(-z)}{[1+\exp(-z)]^2}, \quad F(z;\sigma) = \frac{1-\exp(-z)}{1+\exp(-z)}, \quad z>0.
$$

The main aim of this paper is to propose the estimators of the parameter and reliability function of the HL distribution under progressive type II censoring scheme. First, we derive the MPSE for the parameter and reliability function of HL distribution. And we derive the approximate MPSE for the parameter and reliability function of HL distribution using Talyor series expansion. We also compare the proposed estimators in the sense of the root mean squared error (RMSE) and bias for various progressive type II censored samples.

The rest of the paper is organized as follows. In Section 2, different estimation procedures are discussed and estimators of parameter and reliability function using MPS method and

Taylor series expansion is proposed. In Section 3, provides the real data example and comparison of proposed estimators is conducted using Monte Carlo simulation. Finally concluding remark is presented in Section 4.

2. Estimation

2.1. Maximum product spacings estimation

This section deals with deriving MPSE of the unknown parameter of a HL distribution. Suppose that $X_{1:m:n}$, $X_{2:m:n}$, $\cdots$, $X_{m:m:n}$ denote the observed values of such a progressive type II censored sample. The m and $\boldsymbol{R} = (R_1, R_2, \cdots, R_m)$ are pre-fixed integers satisfying $\sum_{i=1}^{m} R_i + m = n$. Using Ng *et al.* (2012) and Eq (1.1), the product spacings under progressive type II censoring scheme is given by

$$\pounds(\sigma) = K \prod_{i=1}^{m+1} \left[F(x_{i:m:n}) - F(x_{i-1:m:n})\right] \prod_{i=1}^{m} \left[1 - F(x_{i:m:n})\right]^{R_i},$$

where $K = [\prod_{i=1}^{m} \sum_{k=i}^{m} (R_k + 1)]$.

By putting $z_{i:m:n} = x_{i:m:n}/\sigma$, the likelihood function can be written as

$$\pounds(\sigma) = K \prod_{i=1}^{m+1} \left[F(z_{i:m:n}) - F(z_{i-1:m:n})\right] \prod_{i=1}^{m} \left[1 - F(z_{i:m:n})\right]^{R_i}.$$

Hence, the log-likelihood function becomes

$$\ell(\sigma) \propto \sum_{i=1}^{m+1} \log\left[F(z_{i:m:n}) - F(z_{i-1:m:n})\right] + \sum_{i=1}^{m} R_i \log\left[1 - F(z_{i:m:n})\right]. \tag{2.1}$$

Differentiating the log-likelihood function partially with respect to σ and then equating to zero, we have

$$\begin{aligned}\frac{\partial \ell(\sigma)}{\partial \sigma} =& -\frac{1}{2\sigma}\left[2\sum_{i=1}^{m+1} \frac{f(z_{i:m:n})z_{i:m:n} - f(z_{i-1:m:n})z_{i-1:m:n}}{F(z_{i:m:n}) - F(z_{i-1:m:n})}\right. \\ & \left. - \sum_{i=1}^{m} R_i(1 + F(z_{i:m:n}))z_{i:m:n}\right] \\ =& 0.\end{aligned} \tag{2.2}$$

The MPSE of σ is the solution of Eq (2.2). However, solutions for σ is not available. Therefore, we propose to use the Newton-Raphson algorithm to solve it. See for example the work of Gwag and Lee (2018), Lee and Lee (2018) and Kim and Lee (2018). Using the MPSE of σ, say $\hat{\sigma}$, the MPSE of reliability function is obtained as

$$\hat{\Re}(t) = \frac{2\exp(-t/\hat{\sigma})}{1 + \exp(-t/\hat{\sigma})}.$$

2.2. Approximate maximum product spacings estimation

Since the log-likelihood function is very complicated, the Eq (2.1) does not admit an explicit solution for σ. So, we need some approximate likelihood equations which give explicit solutions.

Let

$$\Im_{i:m:n} = F^{-1}(p_{i:m:n}) = -\log\left(\frac{1-p_{i:m:n}}{1+p_{i:m:n}}\right),$$

where

$$p_{i:m:n} = E(U_{i:m:n}) = 1 - \prod_{j=m-i+1}^{m} \frac{j+R_{m-i+1}+\cdots+R_m}{1+j+R_{m-i+1}+\cdots+R_m}.$$

First, we can approximate the functions by Taylor series expansion as follows

$$\frac{f(z_{i:m:n})z_{i:m:n} - f(z_{i-1:m:n})z_{i-1:m:n}}{F(z_{i:m:n}) - F(z_{i-1:m:n})} \simeq \varphi_i + \psi_i z_{i:m:n} + \omega_i z_{i-1:m:n} \tag{2.3}$$

and

$$F(z_{i:m:n})z_{i:m:n} \simeq \nu_{1i} + \kappa_{1i} z_{i:m:n}, \tag{2.4}$$

where

$$\begin{aligned}
\varphi_i =& \frac{f(\Im_{i:m:n})\Im_{i:m:n}^2 p_{i:m:n} - f(\Im_{i-1:m:n})\Im_{i-1:m:n}^2 p_{i-1:m:n}}{p_{i:m:n} - p_{i-1:m:n}}, \\
&+ \left[\frac{f(\Im_{i:m:n})\Im_{i:m:n} - f(\Im_{i-1:m:n})\Im_{i-1:m:n}}{p_{i:m:n} - p_{i-1:m:n}}\right], \\
\psi_i =& \frac{f(\Im_{i:m:n})}{p_{i:m:n} - p_{i-1:m:n}}\left[1 - p_{i:m:n}\Im_{i:m:n}\right. \\
&\left. - \frac{f(\Im_{i:m:n})\Im_{i:m:n} - f(\Im_{i-1:m:n})\Im_{i-1:m:n}}{p_{i:m:n} - p_{i-1:m:n}}\right], \\
\omega_i =& -\frac{f(\Im_{i-1:m:n})}{p_{i:m:n} - p_{i-1:m:n}}\left[1 - p_{i-1:m:n}\Im_{i-1:m:n}\right. \\
&\left. - \frac{f(\Im_{i:m:n})\Im_{i:m:n} - f(\Im_{i-1:m:n})\Im_{i-1:m:n}}{p_{i:m:n} - p_{i-1:m:n}}\right], \\
\nu_{1i} =& -f(\Im_{i:m:n})\Im_{i:m:n}^2, \\
\kappa_{1i} =& f(\Im_{i:m:n})\Im_{i:m:n} + p_{i:m:n}.
\end{aligned}$$

By substituting the Eqs (2.3) and (2.4) into the Eq (2.2), we can approximate the likelihood equation for σ as follows

$$
\begin{aligned}
\frac{\partial \ell(\sigma)}{\partial \sigma} \simeq & -\frac{1}{2\sigma}\left[2\sum_{i=1}^{m+1}\{\varphi_i+\psi_i z_{i:m:n}+\omega_i z_{i-1:m:n}\}\right. \\
& \left. -\sum_{i=1}^{m} R_i z_{i:m:n}-\sum_{i=1}^{m} R_i\{\nu_{1i}+\kappa_{1i} z_{i:m:n}\}\right] \\
=&0.
\end{aligned} \tag{2.5}
$$

Upon solving the Eq (2.5) for σ, we can obtain an approximate MPSE of σ as follows

$$
\hat{\sigma}_{A1}=\frac{A_1}{B_1}, \tag{2.6}
$$

where

$$
\begin{aligned}
A_1 =& \sum_{i=1}^{m}(R_i+\kappa_{1i})x_{i:m:n}-\sum_{i=1}^{m+1}(\psi_i x_{i:m:n}+\omega_i x_{i-1:m:n}) \\
B_1 =& 2\sum_{i=1}^{m+1}\varphi_i-\sum_{i=1}^{m} R_i\nu_i.
\end{aligned}
$$

Next, we can approximate the likelihood function (2.2) by the Eq (2.3) and

$$
F(z_{i:m:n}) \simeq \nu_{2i}+\kappa_{2i} z_{i:m:n}, \tag{2.7}
$$

where

$$
\begin{aligned}
\nu_{2i} =& p_{i:m:n}-f(\Im_{i:m:n})\Im_{i:m:n}, \\
\kappa_{2i} =& f(\Im_{i:m:n}).
\end{aligned}
$$

By substituting the Eqs (2.3) and (2.7) into the Eq (2.2), we can approximate the likelihood equation for σ as follows

$$
\begin{aligned}
\frac{\partial \ell(\sigma)}{\partial \sigma} \simeq & -\frac{1}{2\sigma}\left[2\sum_{i=1}^{m+1}\{\varphi_i+\psi_i z_{i:m:n}+\omega_i z_{i-1:m:n}\}\right. \\
& \left. -\sum_{i=1}^{m} R_i z_{i:m:n}-\sum_{i=1}^{m} R_i\{\nu_{2i}+\kappa_{2i} z_{i:m:n}\}z_{i:m:n}\right] \\
=&0.
\end{aligned} \tag{2.8}
$$

Upon solving the Eq (2.8) for σ, we can obtain an approximate MPSE of σ as follows

$$\hat{\sigma}_{A2} = \frac{-B_2 + \sqrt{B_2^2 - 4A_2C_2}}{2A_2}, \tag{2.9}$$

where

$$\begin{aligned}
A_2 =& 2\sum_{i=1}^{m} \varphi_{i:m:n},\\
B_2 =& 2\sum_{i=1}^{m+1} (\psi_i x_{i:m:n} + \omega_i x_{i-1:m:n}) - \sum_{i=1}^{m} R_i x_{i:m:n} - \sum_{i=1}^{m} R_i \nu_{2i} x_{i:m:n},\\
C_2 =& -\sum_{i=1}^{m} R_i \kappa_{2i} x_{i:m:n}^2.
\end{aligned}$$

Using the approximate MPSEs of σ, say $\hat{\sigma}_{A1}$ and $\hat{\sigma}_{A2}$, the approximate MPSEs of reliability function are obtained as

$$\hat{\Re}_{A1}(t) = \frac{2\exp(-t/\hat{\sigma}_{A1})}{1+\exp(-t/\hat{\sigma}_{A1})} \quad \text{and} \quad \hat{\Re}_{A2}(t) = \frac{2\exp(-t/\hat{\sigma}_{A2})}{1+\exp(-t/\hat{\sigma}_{A2})}. \tag{2.10}$$

3. Illustrative examples and simulation results

3.1. Real example

In order to analyze the real example, we use the proposed estimators in the above section. The real example were from the data on failure times, in minutes, for a specific type of electrical insulation that was subjected to a continuously increasing voltage stress (Lawless, 1982). Gwag and Lee (2018) have examined the goodness-of-fit of the data to HL distribution and they found that the HL distribution fits the data. In this example, we consider the case when the data are progressive type II censored with the following schemes: $n = 12$, $m = 10$ and $R_1 = R_{10} = 1$, $R_2 = \cdots = R_9 = 0$. The observations and censoring scheme are given in Table 3.1. From the Eqs (2.2), (2.6) and (2.9), the MPSE $\hat{\sigma} = 52.8890$, the approximate MPSEs $\hat{\sigma}_{A1} = 52.4426$ and $\hat{\sigma}_{A2} = 52.5595$ are obtained. Also, from the Eqs (2.2) and (2.10), the MPSE $\hat{\Re}(t = 2) = 0.9781$, the approximate MPSEs $\hat{\Re}_{A1}(t = 2) = 0.9809$ and $\hat{\Re}_{A2}(t = 2) = 0.9810$ are obtained.

Table 3.1 The observations and censoring schemes for example

i	1	2	3	4	5	6	7	8	9	10
R_i	1	0	0	0	0	0	0	0	0	1
$x_{i:m:n}$	12.30	21.80	24.40	43.20	46.90	70.70	75.30	98.10	138.60	151.90

3.2. Simulation results

In this section, a Monte Carlo simulation is conducted to compare the performance of MPSE and approximate MPSEs. We consider different n, m and $\boldsymbol{R}$. We have used three different progressive type II censored sampling schemes. First, $R_m = n - m$ and $R_i = 0$ for $i = 1, \cdots, m-1$. Second, $R_1 = n - m$ and $R_i = 0$ for $i = 2, \cdots, m$. Third, $R_1 = R_m = (n-m)/2$ and $R_i = 0$ for $i = 2, \cdots, m-1$. Various progressive type II censoring schemes have been taken into consideration to calculate bias and RMSE of all proposed estimators.

In Tables 3.2 and 3.3, MSEs and biases of all estimates of parameter and reliability function are presented for various choices of n, m and $\boldsymbol{R}$. In general, we observed that the MSEs decrease as the sample size n increases. For a fixed n, the MSEs decrease generally as the progressive type II censored samples size m decreases. In $R_m = n - m$ and $R_i = 0$ for $i = 1, \cdots, m-1$ scheme, and $R_1 = R_m = (n-m)/2$ and $R_i = 0$ for $i = 2, \cdots, m-1$ scheme, we observed that approximate MPSEs are superior to the respective MPSE in terms of MSEs and biases. However, in $R_1 = n - m$ and $R_i = 0$ for $i = 2, \cdots, m$ scheme, we observed that MPSE are superior to the respective approximate MPSEs in terms of MSEs and biases. In particular, approximate MPSE $\hat{\sigma}_{A2}$ and $\hat{\Re}_{A2}(t)$ are better than the corresponding approximate MPSE $\hat{\sigma}_{A1}$ and $\hat{\Re}_{A1}$.

Table 3.2 The relative RMSEs and biases of parameter estimators with MPSE and approximate MPSEs

			RMSE (bias)		
n	m	$\boldsymbol{R}$	$\hat{\sigma}$	$\hat{\sigma}_{A1}$	$\hat{\sigma}_{A2}$
20	18	(0*17,2)	0.2709 (-0.2142)	0.1964 (-0.0621)	0.1963 (-0.0615)
		(2,0*17)	0.1394 (-0.0288)	0.2025 (-0.0854)	0.2024 (-0.0852)
		(1,0*16,1)	0.2110 (-0.1233)	0.1981 (-0.0697)	0.1978 (-0.0683)
	14	(0*13,6)	0.4346 (-0.4152)	0.2218 (-0.0726)	0.2209 (-0.0589)
		(6,0*13)	0.1611 (-0.0156)	0.2431 (-0.1454)	0.2428 (-0.1445)
		(3,0*12,3)	0.3411 (-0.2988)	0.2212 (-0.0730)	0.2208 (-0.0700)
	10	(0*9,10)	0.4406 (-0.4009)	0.2725 (-0.1279)	0.2659 (-0.0720)
		(10,0*9)	0.2186 (0.0087)	0.3437 (-0.2814)	0.3424 (-0.2791)
		(5,0*8,5)	0.4179 (-0.3756)	0.2632 (-0.0850)	0.2632 (-0.0800)
30	28	(0*27,2)	0.2375 (-0.1952)	0.1571 (-0.0477)	0.1571 (-0.0473)
		(2,0*27)	0.1064 (1-.0341)	0.1611 (-0.0616)	0.1611 (-0.0616)
		(1,0*26,1)	0.1772 (-0.1170)	0.1585 (-0.0530)	0.1583 (-0.0524)
	24	(0*23,6)	0.4015 (-0.3886)	0.1686 (-0.0432)	0.1682 (-0.0366)
		(6,0*23)	0.1166 (-0.0306)	0.1778 (-0.0806)	0.1777 (-0.0804)
		(3,0*22,3)	0.2990 (-0.2672)	0.1695 (-0.0497)	0.1694 (-0.0484)
	20	(0*9,20)	0.4849 (-0.4768)	0.1866 (-0.0570)	0.1854 (-0.0307)
		(20,0*9)	0.1343 (-0.0319)	0.2121 (-0.1350)	0.2119 (-0.1342)
		(5,0*18,5)	0.3896 (-0.3716)	0.1851 (-0.0522)	0.1850 (-0.0504)
40	38	(0*37,2)	0.2124 (-0.1764)	0.1388 (-0.0402)	0.1388 (-0.0400)
		(2,0*37)	0.0878 (-0.0322)	0.1417 (-0.0509)	0.1417 (-0.0508)
		(1,0*36,1)	0.1542 (-0.1043)	0.1398 (-0.0446)	0.1397 (-0.0442)
	34	(0*33,6)	0.3702 (-0.3589)	0.1402 (-0.0343)	0.1399 (-0.0305)
		(6,0*33)	0.0937 (-0.0336)	0.1460 (-0.0582)	0.1460 (-0.0581)
		(3,0*32,3)	0.2674 (-0.2391)	0.1414 (-0.0414)	0.1412 (-0.0406)
	30	(0*29,10)	0.4618 (-0.4560)	0.1497 (-0.0352)	0.1493 (-0.0207)
		(10,0*29)	0.1061 (-0.0352)	0.1625 (-0.0830)	0.1624 (-0.0829)
		(5,0*28,5)	0.3567 (-0.3421)	0.1501 (-0.0407)	0.1500 (-0.0397)

Table 3.3 The relative RMSEs and biases of reliability function ($t = 2$) estimators with MPSE and approximate MPSEs

			RMSE (bias)		
n	m	$\boldsymbol{R}$	$\hat{\Re}$	$\hat{\Re}_{A1}$	$\hat{\Re}_{A2}$
20	18	(0*17,2)	0.1147 (-0.0915)	0.0814 (-0.0285)	0.0814 (-0.0283)
		(2,0*17)	0.0593 (-0.0132)	0.0846 (-0.0381)	0.0845 (-0.0380)
		(1,0*16,1)	0.0898 (-0.0533)	0.0824 (-0.0316)	0.0822 (-0.0310)
	14	(0*13,6)	0.1763 (-0.1700)	0.0914 (-0.0333)	0.0905 (-0.0277)
		(6,0*13)	0.0676 (-0.0081)	0.1019 (-0.0630)	0.1018 (-0.0626)
		(3,0*12,3)	0.1421 (-0.1256)	0.0912 (-0.0334)	0.0909 (-0.0322)
	10	(0*9,10)	0.1749 (-0.1961)	0.1111 (-0.0558)	0.1065 (-0.0337)
		(10,0*9)	0.0883 (0.0004)	0.1404 (-0.1170)	0.1399 (-0.1161)
		(5,0*8,5)	0.1680 (-0.1531)	0.1061 (-0.0388)	0.1059 (-0.0368)
30	28	(0*27,2)	0.1017 (-0.0839)	0.0658 (-0.0218)	0.0658 (-0.0217)
		(2,0*27)	0.0457 (-0.0150)	0.0678 (-0.0276)	0.0678 (-0.0276)
		(1,0*26,1)	0.0761 (-0.0506)	0.0665 (-0.0240)	0.0664 (-0.0238)
	24	(0*23,6)	0.1666 (-0.1622)	0.0701 (-0.0202)	0.0697 (-0.0175)
		(6,0*23)	0.0498 (-0.0136)	0.0750 (-0.0357)	0.0750 (-0.0356)
		(3,0*22,3)	0.1268 (-0.1139)	0.0707 (-0.0229)	0.0706 (-0.0223)
	20	(0*9,20)	0.1942 (-0.1921)	0.0774 (-0.0262)	0.0759 (-0.0155)
		(20,0*9)	0.0571 (-0.0143)	0.0899 (-0.0584)	0.0898 (-0.0582)
		(5,0*18,5)	0.1616 (-0.1553)	0.0767 (-0.0242)	0.0766 (-0.0235)
40	38	(0*37,2)	0.0914 (-0.0760)	0.0584 (-0.0184)	0.0584 (-0.0183)
		(2,0*37)	0.0378 (-0.0140)	0.0599 (-0.0229)	0.0599 (-0.0229)
		(1,0*36,1)	0.0665 (-0.0450)	0.0589 (-0.0202)	0.0589 (-0.0201)
	34	(0*33,6)	0.1558 (-0.1517)	0.0589 (-0.0159)	0.0586 (-0.0144)
		(6,0*33)	0.0404 (-0.0147)	0.0619 (-0.0259)	0.0619 (-0.0259)
		(3,0*32,3)	0.1144 (-0.1026)	0.0596 (-0.0189)	0.0595 (-0.0185)
	30	(0*29,10)	0.1883 (-0.1868)	0.0627 (-0.0165)	0.0620 (-0.0105)
		(10,0*29)	0.0455 (-0.0154)	0.0692 (-0.0365)	0.0692 (-0.0364)
		(5,0*28,5)	0.1504 (-0.1449)	0.0630 (-0.0188)	0.0629 (-0.0184)

4. Conclusions

In this paper, we consider the MPSE and approximate MPSE of the parameter and reliability function of a HL distribution under progressive type II censoring scheme. We observed that the MSEs decrease as the sample size and progressive type II censored sample size increases. In last censoring case ($R_m = n - m$ and $R_i = 0$ for $i = 1, \cdots, m-1$) and doubly censoring case ($R_1 = R_m = (n-m)/2$ and $R_i = 0$ for $i = 2, \cdots, m-1$), we observed that approximate MPSEs are superior to the respective MPSE in terms of MSEs and biases. In first censoring case ($R_1 = n - m$ and $R_i = 0$ for $i = 2, \cdots, m$), we observed that MPSE are superior to the respective approximate MPSEs in terms of MSEs and biases. Also, approximate MPSE $\hat{\sigma}_{A2}$ and $\hat{\Re}_{A2}(t)$ are better than the corresponding approximate MPSE $\hat{\sigma}_{A1}$ and $\hat{\Re}_{A1}$. Although we focused on the parameter and reliability estimate of the HL distribution based on progressive type II censoring scheme, approximate MPSE of the parameter and reliability from other distributions based on progressive type II censoring scheme is of potential interest in future research.

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