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Estimating the parameter of the exponentiated half-logistic distribution under generalized type II hybrid censoring scheme†

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Abstract

In this paper, we consider the parameter for the exponentiated half logistic distribution (ExHfLg) when data are generalized type II hybrid censored (GenTy2HC) samples. The parameter for the ExHfLg is estimated by the Bayesian method. We consider conjugate priors (gamman and quasi prior) and corresponding posterior distributions are obtained. We also obtain the maximum likelihood estimator (MLE) of the parameter under the GenTy2HC samples. We compare the proposed estimators in the terms of the mean square error and bias. Finally, a real data set has been analysed for illustrative purpose.

Keywords: Bayesian estimation, exponentiated half-logistic distribution, gamma and quasi prior distributions, generalized type II hybrid censoring, half-logistic distribution, maximum likelihood estimation, squared error and Linex loss functions.

1. Introduction

Consider a life testing experiment in which n units are put on test. Assume that the life times of n units are independent and identically distributed (iid) as ExHfLg with the cumulative density function (cdf) and the probability density function (pdf)

$$F(x;\theta) = \left[\frac{1-\exp(-x/\sigma)}{1+\exp(-x/\sigma)}\right]^{\theta}, \tag{1.1}$$

$$f(x;\theta) = \frac{2\theta\exp(-x/\sigma)}{\sigma[1+\exp(-x/\sigma)]}\left[\frac{1-\exp(-x/\sigma)}{1+\exp(-x/\sigma)}\right]^{\theta-1},\ x>0,\theta>0, \tag{1.2}$$

where $\theta > 0$ is shape parameter.

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Inferences for the ExHfLg were discussed by several authors. Alotaibi *et al.* (2021) considered bivariate ExHfLg and its properties and application. Lee (2021) derived the MLE and Bayesian estimator of the parameter in a ExHfLg based on generalized type I hybrid censored samples. Hassan *et al.* (2022) considered the Bayesian and non-Bayesian inference for unit-ExHfLg. Rao and Kirigiti (2023) considered the repetitive sampling inspection plan for cancer patients using ExHfLg under indeterminacy. Srinivas Rao *et al.* (2023) considered the multiple dependent state repetitive sampling plans for ExHfLg. Song *et al.* (2022) derived the parameter estimation of ExHfLg for left-truncated and right-censored data.

The detail description of the GenTy2HC scheme is described as follows. The integer $r \in \{1, 2, ..., n\}$ is pre-fixed. $T_1 \in (0, \infty)$, $T_2 \in (0, \infty)$ are a pre-fixed time points ($T_1 < T_2$). If the rth failure occurs before time T_1, terminate the experiment at $X_{d_1:n}$; if the rth failure occurs between T_1 and T_2, terminate at $X_{r:n}$; if the rth failures occur after T_2, terminate at $X_{d_2:n}$ (See Chandrasekar *et al.* (2004)). Here, assume that d_i denote the number of observed failures up to time T_i ($i = 1, 2$). In this scheme, we have one of the following three types of observations;

Case 1 : $X_{1:n}, X_{2:n}, ..., X_{r:n}, ..., X_{d_1:n}$, if $X_{r:n} < X_{d_1:n} < T_1$,
Case 2 : $X_{1:n}, X_{2:n}, ..., X_{r:n}$, if $T_1 < X_{r:n} < T_2$,
Case 3 : $X_{1:n}, X_{2:n}, ..., X_{d_2:n}$, if $X_{d_2:n} < T_2 < X_{r:n}$.

A schematic representation of the GenTy2HC scheme is presented in Figure 1.1.

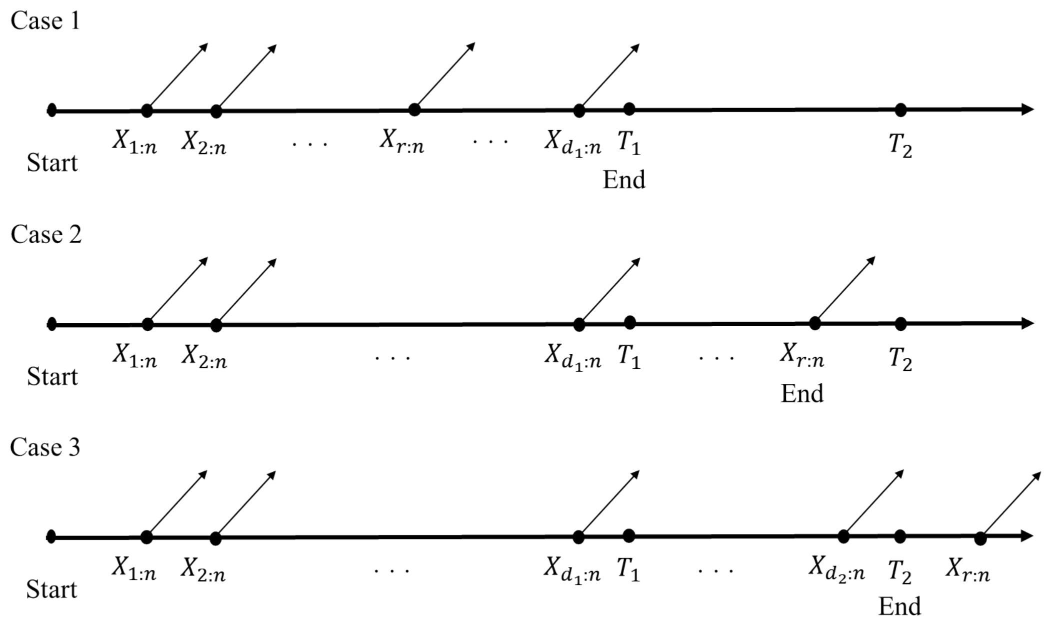

Figure 1.1 Schematic illustration of GenTy2HC scheme

This study is two aims. The 1st is to consider the MLE of the parameter when the samples are GenTy2HC scheme. The second is to consider the Bayesian estimation for the parameter under square error loss function (SqL) and linex loss function (LxL).

The rest of this paper is organized as follows. In Section 2, we describe the computation of the MLE of the parameter based on the GenTy2HC scheme. In Section 3, Bayesian estimators of the parameter under the SqL and LxL are derived. In Section 4, the proposed estimators that are compared by performing the Monte Carlo simulation is presented, and Section 5 concludes.

2. Maximum likelihood estimation

Assume that the failure times of the units are the ExHfLg. The likelihood functions under GenTy2HC scheme are as follows.

$$\text{Case 1}: L_1(\theta) \propto \frac{n!}{(n-d_1)!} \prod_{i=1}^{d_1} f(x_{i:n}) \left[1 - F(T_1)\right]^{n-d_1},$$

$$\text{Case 2}: L_2(\theta) \propto \frac{n!}{(n-r)!} \prod_{i=1}^{r} f(x_{i:n}) \left[1 - F(x_{r:n})\right]^{n-r},$$

$$\text{Case 3}: L_3(\theta) \propto \frac{n!}{(n-d_2)!} \prod_{i=1}^{d_2} f(x_{i:n}) \left[1 - F(T_2)\right]^{n-d_2}.$$

Above three cases can be combined and be represented as

$$L(\theta) \propto \frac{n!}{(n-\tau)!} \prod_{i=1}^{\tau} f(x_{i:n}) \left[1 - F(\xi)\right]^{n-\tau},$$

where $\tau = d_1$ and $\xi = T_1$ for Case I, $\tau = r$ and $\xi = x_{r:n}$ for Case II, and $\tau = d_2$ and $\xi = T_2$ for Case III. Then, from (1.1) and (1.2), the likelihood function under GenTy2HC scheme are as follows.

$$L(\theta) \propto \theta^{\tau} \prod_{i=1}^{\tau} \left[\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right]^{\theta-1} \left[1 - \left\{\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right\}^{\theta}\right]^{n-\tau}, \qquad (2.1)$$

From (2.1), the log-likelihood function can be expressed as

$$\ell(\theta) \propto \tau \log\theta + (\theta-1)\sum_{i=1}^{\tau} \log\left[\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right] + (n-\tau)\log\left[1 - \left\{\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right\}^{\theta}\right].$$

On differentiating the log-likelihood function with the respect to parameter (θ) and equating to zero, we obtain the estimating equation,

$$\frac{\partial \ell(\theta)}{\partial\theta} = \frac{\tau}{\theta} + \sum_{i=1}^{\tau} \log\left[\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right] - (n-\tau)\frac{\log\left[\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right]}{1-\left[\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right]^{\theta}} = 0.$$

This equation is in implicit form, so it may be subsequently solved with a numerical method.

3. Bayesian estimation

In Bayesian method, first of all, we consider SqL function which is symmetric. But in life testing problems, the nature of losses are not always symmetric. Therefore, we also consider LxL which is asymmetric. Since based on the GenTy2HC scheme is a random variable, first of all, we consider the gamma prior distribution for that were used as

$$\pi_1(\theta;\alpha,\beta) \propto \theta^{\alpha-1}\exp(-\beta\theta), \quad \alpha>0,\ \beta>0. \tag{3.1}$$

By combining (2.1) with (3.1), the joint density function of θ and $\boldsymbol{X}$ is given by

$$\pi_1(\theta,\boldsymbol{X}) \propto \theta^{\tau+\alpha-1}\zeta_1(\beta,0),$$

where

$$\begin{aligned}\zeta_1(a_1,a_2) =& \exp\left[-\theta\left\{a_1+a_2-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right\}\right]\\ &\times\left[1-\left\{\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right\}^{\theta}\right]^{n-\tau}.\end{aligned}$$

Then,

$$\begin{aligned}\int_0^\infty \pi_1(\theta,\boldsymbol{X})d\theta =& \int_0^\infty \theta^{\tau+\alpha-1}\zeta_1(\beta,0)d\theta\\ =& \int_0^\infty \theta^{\tau+\alpha-1}\exp\left[-\theta\left\{\beta-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right\}\right]\\ &\times\sum_{j=0}^{\tau}(-1)^j\binom{n-z}{j}\left\{\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right\}^{j\theta}d\theta\\ =& \sum_{j=0}^{\tau}(-1)^j\binom{n-\tau}{j}\int_0^\infty \theta^{\tau+\alpha-1}\exp\left[-\theta\left\{\beta-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right.\right.\\ &\left.\left.-j\log\left(\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right)\right\}\right]d\theta\\ =& \Gamma(\tau+\alpha)\zeta_2(\beta,0,\tau+\alpha),\end{aligned}$$

where

$$\begin{aligned}\zeta_2(a_1,a_2,a_3) =& \sum_{j=0}^{\tau}(-1)^j\binom{n-\tau}{j}\left[a_1+a_2-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right.\\ &\left.-j\log\left(\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right)\right]^{-a_3}.\end{aligned}$$

Therefore, the posterior density function of θ is given by

$$\pi_1(\theta|\boldsymbol{X}) = \frac{\theta^{\tau+\alpha-1}\zeta_1(\beta,0)}{\Gamma(\tau+\alpha)\zeta_2(\beta,0,\tau+\alpha)}.$$

For the situation where no prior information about the parameter θ is available, one may use the quasi density as given by

$$\pi_2(\theta;\gamma) = \frac{1}{\theta^\gamma}, \quad \gamma > 0. \tag{3.2}$$

By combining (2.1) with (3.2), the joint density function of θ and $\boldsymbol{X}$ is given by

$$\pi_2(\theta, \boldsymbol{X}) \propto \theta^{\tau-\gamma}\zeta_1(0,0).$$

Further, the posterior density function of θ is given by

$$\pi_2(\theta|\boldsymbol{X}) = \frac{\theta^{\tau-\gamma}\zeta_1(0,0)}{\Gamma(\tau-\gamma+1)\zeta_2(0,0,\tau-\gamma+1)},$$

where

$$\begin{aligned}
\int_0^\infty \pi_2(\theta,\boldsymbol{X})d\theta =& \int_0^\infty \theta^{\tau-\gamma}\zeta_1(0,0)d\theta \\
=& \int_0^\infty \theta^{\tau-\gamma}\exp\left[-\theta\left\{-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right\}\right] \\
&\times\sum_{j=0}^{\tau}(-1)^j\binom{n-\tau}{j}\left\{\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right\}^{j\theta}d\theta \\
=& \sum_{j=0}^{\tau}(-1)^j\binom{n-\tau}{j}\int_0^\infty \theta^{\tau-\gamma}\exp\left[-\theta\left\{-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right.\right. \\
&\left.\left.-j\log\left(\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right)\right\}\right]d\theta \\
=& \Gamma(\tau-\gamma+1)\sum_{j=0}^{\tau}(-1)^j\binom{n-\tau}{j}\left[-\sum_{i=1}^{\tau}\log\left(\frac{1-\exp(-x_{i:n}/\sigma)}{1+\exp(-x_{i:n}/\sigma)}\right)\right. \\
&\left.-j\log\left(\frac{1-\exp(-\xi/\sigma)}{1+\exp(-\xi/\sigma)}\right)\right]^{-(\tau-\gamma+1)} \\
=& \Gamma(\tau-\gamma+1)\zeta_2(0,0,\tau-\gamma+1).
\end{aligned}$$

Under SqL and gamma prior distribution, the Bayesian estimator of θ is the mean of the posterior density given by

$$\hat{\theta}_{s_1} = \int_0^\infty \theta\pi_1(\theta|\boldsymbol{X})d\theta = \frac{(\tau+\alpha)\zeta_2(\beta,0,\tau+\alpha+1)}{\zeta_2(\beta,0,\tau+\alpha)}.$$

Under SqL and quasi prior distribution, the Bayesian estimator of θ is the mean of the posterior density given by

$$\hat{\theta}_{s_2} = \int_0^\infty \theta\pi_2(\theta|\boldsymbol{X})d\theta = \frac{(\tau-\gamma+1)\zeta_2(0,0,\tau-\gamma+2)}{\zeta_2(0,0,\tau-\gamma+1)}.$$

Under LxL and gamma prior distribution, the Bayesian estimator of θ is given by

$$\hat{\theta}_{l_1} = -\frac{1}{h}\log E(e^{-h\theta}) = -\frac{1}{h}\log\frac{\zeta_2(\beta, h, \tau+\alpha)}{\zeta_2(\beta, 0, \tau+\alpha)},$$

where h is the scale parameter of LxL,

$$\begin{aligned}
E(e^{-h\theta}) =& \int_0^\infty e^{-h\theta}\pi_1(\theta|\boldsymbol{X})d\theta \\
=& \frac{1}{\Gamma(\tau+\alpha)\zeta_2(\beta, 0, \tau+\alpha)}\int_0^\infty \theta^{\tau+\alpha-1}\zeta_1(\beta, h)d\theta \\
=& \frac{\zeta_2(\beta, h, \tau+\alpha)}{\zeta_2(\beta, 0, \tau+\alpha)}
\end{aligned}$$

Under LxL and quasi prior distribution, the Bayesian estimator of θ is given by

$$\hat{\theta}_{l_2} = -\frac{1}{h}\log E(e^{-h\theta}) = -\frac{1}{h}\log\frac{\zeta_2(0, h, \tau-\gamma+1)}{\zeta_2(0, 0, \tau-\gamma+1)},$$

where

$$\begin{aligned}
E(e^{-h\theta}) =& \int_0^\infty e^{-h\theta}\pi_2(\theta|\boldsymbol{X})d\theta \\
=& \frac{1}{\Gamma(\tau-\gamma+1)\zeta_2(0, 0, \tau-\gamma+1)}\int_0^\infty \theta^{\tau-\gamma}\zeta_1(0, h)d\theta \\
=& \frac{\zeta_2(0, h, \tau-\gamma+1)}{\zeta_2(0, 0, \tau-\gamma+1)}.
\end{aligned}$$

4. Illustrative example and simulation results

4.1. Illustrative example

Mann and Fertig (1973) give failure times to airplane components subjected to a life test. The samples are GenTy2HC samples : 13 components were placed on test. For this samples, Lee (2021) indicated that the ExHfLg provides a satisfactory fit. In this sample, we assume that the underlying distribution of this data is the ExHfLg based on the three GenTy2HC schemes (Case 1: $r = 8$, T_1= 2.0 and $T_2 = 10$; Case 2: $r = 4$, T_1= 2.0 and $T_2 = 8$; Case 3: $r = 4$, T_1= 2.0 and $T_2 = 8$). For Case 1, the Bayesian estimators under gamma prior distribution of $\theta_{s_1} = 1.672955$ and $\theta_{l_1} = 1.620997$ are obtained. Also, the Bayesian estimators under quasi prior distribution of $\theta_{s_2} = 1.932297$ and $\theta_{l_2} = 1.87217$ are obtained. And the MLE of $\theta = 2.030625$ is obtained. For Case 1, the Bayesian estimators under gamma prior distribution of $\theta_{s_1} = 1.672955$ and $\theta_{l_1} = 1.620997$ are obtained. Also, the Bayesian estimators under quasi prior distribution of $\theta_{s_2} = 1.932297$ and $\theta_{l_2} = 1.87217$ are obtained. And the MLE of $\theta = 2.030625$ is obtained. For Case 3, the Bayesian estimators under gamma prior distribution of $\theta_{s_1} = 1.672955$ and $\theta_{l_1} = 1.620997$ are obtained. Also, the Bayesian estimators under quasi prior distribution of $\theta_{s_2} = 1.932297$ and $\theta_{l_2} = 1.87217$ are obtained. And the MLE of $\theta = 2.030625$ is obtained.

4.2. Simulation results

To compare the performance of the MLE and Bayesian estimators of θ under the SqL and LxL, we simulated the mean squared errors and biases of proposed estimators through Monte Carlo simulation method. We consider various n, r, T_1 and T_2.

Table 4.1 The relative mean squared errors and biases of proposed estimators (under gamma prior distribution)

				MSE (bias)				
n	T_1	T_2	r	$\hat{\theta}$	$\hat{\theta}_{S_1}$	$\hat{\theta}_{L_1}$ $(h=-1.5)$	$\hat{\theta}_{L_1}$ $(h=-.5)$	$\hat{\theta}_{L_1}$ $(h=.5)$
20	.8	1.2	14	.0419(.1071)	.0191(-.1118)	.0179(-.1030)	.0187(-.1089)	.0195(-.1145)
			16	.0382(.0890)	.0168(-.1002)	.0158(-.0912)	.0165(-.0971)	.0172(-.1028)
			18	.0382(.0851)	.0161(-.0968)	.0151(-.0876)	.0157(-.0936)	.0164(-.0993)
		1.5	14	.0326(.0944)	.0181(-.1027)	.0172(-.0937)	.0178(-.0996)	.0185(-.1053)
			16	.0258(.0655)	.0146(-.0805)	.0140(-.0710)	.0144(-.0771)	.0148(-.0830)
			18	.0252(.0564)	.0131(-.0709)	.0126(-.0611)	.0129(-.0673)	.0133(-.0733)
	1	1.5	14	.0312(.0843)	.0167(-.0967)	.0158(-.0877)	.0164(-.0937)	.0170(-.0994)
			16	.0259(.0641)	.0143(-.0794)	.0137(-.0701)	.0141(-.0762)	.0145(-.0821)
			18	.0252(.0564)	.0131(-.0709)	.0127(-.0613)	.0129(-.0675)	.0133(-.0735)
		1.8	14	.0296(.0818)	.0166(-.0941)	.0158(-.0851)	.0163(-.0911)	.0169(-.0969)
			16	.0221(.0558)	.0138(-.0685)	.0134(-.0588)	.0136(-.0651)	.0139(-.0712)
			18	.0204(.0427)	.0121(-.0508)	.0121(-.0405)	.0120(-.0470)	.0121(-.0532)
30	.8	1.2	21	.0231(.0878)	.0187(-.1204)	.0177(-.1149)	.0184(-.1185)	.0191(-.1220)
			24	.0203(.0693)	.0160(-.1077)	.0150(-.1021)	.0156(-.1058)	.0162(-.1093)
			27	.0203(.0666)	.0154(-.1052)	.0145(-.0996)	.0151(-.1033)	.0157(-.1069)
		1.5	21	.0196(.0797)	.0176(-.1133)	.0166(-.1078)	.0173(-.1115)	.0179(-.1150)
			24	.0141(.0493)	.0129(-.0882)	.0122(-.0823)	.0127(-.0861)	.0131(-.0898)
			27	.0138(.0416)	.0115(-.0793)	.0109(-.0733)	.0112(-.0772)	.0117(-.0809)
	1	1.5	21	.0188(.0709)	.0161(-.1079)	.0153(-.1024)	.0158(-.1061)	.0164(-.1097)
			24	.0142(.0485)	.0127(-.0876)	.0121(-.0818)	.0125(-.0856)	.0130(-.0893)
			27	.0138(.0416)	.0115(-.0793)	.0109(-.0734)	.0113(-.0772)	.0117(-.0810)
		1.8	21	.0185(.0701)	.0160(-.1068)	.0151(-.1013)	.0157(-.1050)	.0163(-.1086)
			24	.0124(.0421)	.0117(-.0779)	.0111(-.0719)	.0115(-.0758)	.0119(-.0796)
			27	.0113(.0294)	.0095(-.0593)	.0091(-.0530)	.0093(-.0570)	.0096(-.0609)
40	.8	1.2	28	.0186(.0833)	.0185(-.1233)	.0176(-.1193)	.0182(-.1219)	.0187(-.1245)
			32	.0165(.0651)	.0154(-.1101)	.0146(-.1060)	.0151(-.1087)	.0156(-.1114)
			36	.0165(.0634)	.0150(-.1085)	.0142(-.1043)	.0147(-.1070)	.0152(-.1097)
		1.5	28	.0159(.0765)	.0174(-.1173)	.0167(-.1133)	.0172(-.1160)	.0177(-.1186)
			32	.0112(.0453)	.0121(-.0901)	.0115(-.0858)	.0119(-.0886)	.0123(-.0913)
			36	.0110(.0389)	.0107(-.0823)	.0102(-.0779)	.0105(-.0808)	.0108(-.0835)
	1	1.5	28	.0152(.0685)	.0161(-.1122)	.0153(-.1082)	.0158(-.1109)	.0163(-.1135)
			32	.0112(.0449)	.0120(-.0898)	.0114(-.0855)	.0118(-.0833)	.0122(-.0911)
			36	.0110(.0389)	.0107(-.0823)	.0102(-.0780)	.0105(-.0808)	.0109(-.0836)
		1.8	28	.0150(.0678)	.0159(-.1114)	.0152(-.1074)	.0157(-.1101)	.0162(-.1127)
			32	.0098(.0390)	.0109(-.0807)	.0104(-.0764)	.0107(-.0792)	.0110(-.0820)
			36	.0089(.0268)	.0084(-.0622)	.0081(-.0576)	.0083(-.0605)	.0085(-.0634)

Table 4.2 The relative mean squared errors and biases of proposed estimators (under quasi prior distribution)

				MSE (bias)				
n	T_1	T_2	r	$\hat{\theta}$	$\hat{\theta}_{S_2}$	$\hat{\theta}_{L_2}$ $(h=-1.5)$	$\hat{\theta}_{L_2}$ $(h=-.5)$	$\hat{\theta}_{L_2}$ $(h=.5)$
20	.8	1.2	14	.0419(.1071)	.0210(-.1201)	.0196(-.1114)	.0205(-.1172)	.0214(-.1228)
			16	.0382(.0890)	.0184(-.1082)	.0172(-.0992)	.0180(-.1051)	.0188(-.1108)
			18	.0382(.0851)	.0176(-.1047)	.0165(-.0957)	.0172(-.1016)	.0180(-.1073)
		1.5	14	.0326(.0944)	.0198(-.1107)	.0187(-.1018)	.0194(-.1077)	.0201(-.1134)
			16	.0258(.0655)	.0158(-.0881)	.0151(-.0786)	.0155(-.0847)	.0161(-.0906)
			18	.0252(.0564)	.0142(-.0784)	.0136(-.0686)	.0139(-.0748)	.0144(-.0807)
	1	1.5	14	.0312(.0843)	.0182(-.1046)	.0172(-.0957)	.0178(-.1017)	.0186(-.1074)
			16	.0259(.0641)	.0155(-.0870)	.0148(-.0777)	.0152(-.0838)	.0158(-.0897)
			18	.0252(.0564)	.0142(-.0783)	.0136(-.0687)	.0139(-.0750)	.0144(-.0809)
		1.8	14	.0296(.0818)	.0181(-.1020)	.0172(-.0930)	.0178(-.0990)	.0185(-.1048)
			16	.0221(.0558)	.0148(-.0758)	.0143(-.0662)	.0146(-.0725)	.0149(-.0785)
			18	.0204(.0427)	.0128(-.0578)	.0127(-.0476)	.0127(-.0540)	.0129(-.0602)
30	.8	1.2	21	.0231(.0878)	.0200(-.1259)	.0190(-.1205)	.0197(-.1241)	.0204(-.1276)
			24	.0203(.0693)	.0171(-.1130)	.0161(-.1075)	.0168(-.1111)	.0174(-.1147)
			27	.0203(.0666)	.0165(-.1105)	.0156(-.1049)	.0162(-.1086)	.0168(-.1122)
		1.5	21	.0196(.0797)	.0188(-.1188)	.0178(-.1132)	.0184(-.1169)	.0191(-.1204)
			24	.0141(.0493)	.0138(-.0933)	.0131(-.0874)	.0135(-.0912)	.0140(-.0949)
			27	.0138(.0416)	.0123(-.0843)	.0116(-.0783)	.0120(-.0821)	.0125(-.0859)
	1	1.5	21	.0188(.0709)	.0173(-.1133)	.0163(-.1077)	.0170(-.1114)	.0176(-.1150)
			24	.0142(.0485)	.0136(-.0926)	.0129(-.0868)	.0134(-.0906)	.0139(-.0944)
			27	.0138(.0416)	.0123(-.0843)	.0116(-.0784)	.0120(-.0822)	.0125(-.0860)
		1.8	21	.0185(.0701)	.0171(-.1121)	.0162(-.1066)	.0168(-.1103)	.0174(-.1139)
			24	.0124(.0421)	.0124(-.0828)	.0118(-.0769)	.0122(-.0808)	.0126(-.0846)
			27	.0113(.0294)	.0100(-.0640)	.0096(-.0577)	.0099(-.0617)	.0101(-.0656)
40	.8	1.2	28	.0186(.0833)	.0195(-.1274)	.0186(-.1235)	.0192(-.1261)	.0198(-.1287)
			32	.0165(.0651)	.0162(-.1142)	.0155(-.1101)	.0160(-.1128)	.0165(-.1154)
			36	.0165(.0634)	.0158(-.1125)	.0151(-.1084)	.0156(-.1111)	.0161(-.1137)
		1.5	28	.0159(.0765)	.0184(-.1214)	.0176(-.1174)	.0181(-.1201)	.0186(-.1227)
			32	.0112(.0453)	.0128(-.0939)	.0121(-.0896)	.0125(-.0924)	.0129(-.0952)
			36	.0110(.0389)	.0113(-.0861)	.0107(-.0817)	.0111(-.0845)	.0115(-.0873)
	1	1.5	28	.0152(.0685)	.0170(-.1163)	.0162(-.1123)	.0167(-.1149)	.0172(-.1176)
			32	.0112(.0449)	.0127(-.0936)	.0121(-.0893)	.0125(-.0921)	.0129(-.0949)
			36	.0110(.0389)	.0113(-.0861)	.0107(-.0817)	.0111(-.0846)	.0115(-.0873)
		1.8	28	.0150(.0678)	.0168(-.1154)	.0161(-.1114)	.0166(-.1141)	.0171(-.1167)
			32	.0098(.0390)	.0115(-.0844)	.0109(-.0801)	.0113(-.0829)	.0116(-.0857)
			36	.0089(.0268)	.0089(-.0657)	.0085(-.0612)	.0087(-.0641)	.0090(-.0670)

The GenTy2HC scheme from the ExHfLg are generated for sample size $n = 20, 30, 40$ and various GenTy2HC scheme. Using this generated data, the mean squared errors and biases of the Bayesian estimators of θ under the SqL and the LxL are simulated by the Monte Carlo method based on 1,000 times for sample size $n = 20, 30, 40$ and various GenTy2HC scheme. In each cases, we take $\theta = 0.5$, and we replicate the process 10,000 times. The associated MLE is computed using a Newton-Raphson method. All Bayesian estimates are calculated with respect to the gamma prior ($\alpha = \beta = 0.5$) distribution and quasi prior ($\gamma = 1$) distribution. Bayesian estimates of parameter are obtained with respect to SqL and LxL. Under LxL, Bayesian estimates are obtained for $h = -1.5, -0.5$ and 0.5. And, various schemes have been taken into consideration to calculate bias and mean squared error (MSE) of all estimates. The simulation results under the gamma prior and the quasi prior are given in Tables 4.1 and 4.2, respectively.

As the n and r increases, the mean squared error and bias of the estimates decreases. As the times T_1 and T_2 increases, the mean squared error and bias of the estimates decreases. The MLE of θ is compared with Bayesian estimators under the SqL and the LxL in terms of mean squared error and bias. The computation of Bayesian estimators is better than MLE. The Bayesian estimators under the LxL with $h = .5$ show an overall better performance than their corresponding MLE and Bayesian estimators under the SqL.

5. Conclusions

In this paper, we consider the parameter for the ExHfLg when data are GenTy2HC samples. The parameter for the ExHfLg is estimated by the Bayesian method. We consider conjugate priors (gamma prior and quasi prior distributions) and corresponding posterior distributions are obtained. We also obtain the maximum likelihood estimator of the parameter under the GenTy2HC samples. The MLE of parameter is compared with Bayesian estimators under the SqL and the LxL in terms of estimated mean squared error and bias. The computation of Bayesian estimators is better than MLE. The Bayesian estimators under the LxL with $h = .5$ show an overall better performance than their corresponding MLE and Bayesian estimators under the SqL. Although we focused on the parameter estimate of the ExHfLg based on GenTy2HC scheme, estimation of the parameter from other distributions based on GenTy2HC scheme is of potential interest in future research.

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